MATH 120A Prep: Proofs

1. For the following statements decide which proof technique is best to use to prove the statement and then write down the assumptions and conclusions you would need for the proof. You do not have to write the entire proof.

   (a) Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$. Show if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$.

   (b) Prove there are infinitely many primes.

2. For the following statements decide which proof technique is best to use to prove the statement, then write out the proof.

   (a) Prove for all positive integers $n \geq 4$ that $3^n \geq n^3$.

   (b) Prove that $1 + \sqrt{2}$ is a root of the polynomial $x^2 - 2x - 1$.